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CONTENTS

On fractal self-organization of the financial time series

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Abstract. Time series of five financial indexes daily returns were analyzed by means of multifractal and recurrence quantification analysis (RQA) methods. It is shown that a financial crisis in 2008 year is accompanied with the increase in determinism and fractal self-organization. Such regularity is noted as analogous to other nonlinear systems behavior in catastrophic situations. At the same time, the global Hürst coefficient is minimal during the crises instead of maximum for physical systems.

Keywords: nonlinear dynamical systems, multifractals, recurrence quantification analysis, catastrophes.

1. Introduction

Recently, a certain amount of data have appeared that complex systems of different physical nature behave in the same way in catastrophic situations. The multifractal singularity spectra (SS) of dynamic variables are significantly narrowed, the global Hurst coefficients increase, and the determinism of the process calculated from the time dependences of the quantitative characteristics of recurrent analysis (RQA) increases when approaching a catastrophic event. So, the aim of the research was to find out whether these patterns are present in systems of quite a different nature: financial time series of stock indices. Presence of the same peculiarities in very different systems suggests that these peculiarities are not connected with the details of the system elements interaction but rather with the character of these elements collectivization.

2. Literature Review

The multifractal singularity spectra of the surface profiles of amorphous alloys, measured by probe microscopy, are transformed during mechanical loading and narrow before the destruction of the material [1]. Narrowing of the SS was also observed during the crystallization phase transition in the NiTi alloy [2]. It is known that strong earthquakes and rock bursts are preceded by a decrease in the fractal dimension (an increase in the Hurst coefficient) [3], as well as a decrease in the width of the SS seismic time series [4]. Same effects were noted in the time series of acoustic emission during the formation of a main crack in laboratory samples [5-6]. Multifractal spectra of heart rate for the healthy people are wider, and the Hurst coefficients are lower than for the people suffering from heart disease (chronic heart failure or decreased systolic function of the left ventricle) [7]. Epileptic seizures in children are accompanied by an increase in the determinism of the process, manifested by an increase in the parameters of recurrent plots associated with diagonal lines, as well as an increase in global Hurst coefficients (an increase in persistence) [8]. It was suggested ([8]) that such behavior is associated with the fractal self-organization of systems due to the collectivization of their structural elements at different scales.

3. Data and Methodology

In this paper, we study the time dependences of the SS and the RQA characteristics in the systems of a completely different type –the time series of the S&P500, DJI, Nasdaq-100, DAX and Nikkei 225 stock indices from 1990 to 2019. In particular, it is of interest to study how the multifractal characteristics of these indices and the numerical parameters of RQA behaved during the financial crisis of 2008 and whether they have the patterns described above. It should be noted that there are many papers devoted to the study of multifractal characteristics of financial markets, but these investigations, as a rule, pursued other goals (mainly the study of market efficiency and prediction of its volatility) (see review [9]).

We studied the time series of daily closing prices of stock indices. The daily returns r_i were chosen as a random variable:

$$r_i = \frac{C_i - C_{i-1}}{C_{i-1}} = \frac{\Delta_i}{C_{i-1}}, \tag{1}$$

where C_i and C_{i-1} are the close prices of i of that day and $i-1$ of the day, Δ_i is Bachelier's absolute return. Note that with a small change in daily prices for developed markets, this value practically coincides with the logarithmic return:

$$r\log_i = \ln\frac{C_i}{C_{i-1}} = \ln\left(1+\frac{\Delta_i}{C_{i-1}}\right) \approx \frac{\Delta_i}{C_{i-1}} \tag{2}$$

We used the p-leaders method developed in [10,11] to determine the multifractal characteristics. The p-leaders method is a generalization of the wavelet leaders method developed earlier by the same authors [12]. P-leaders at a given scale level consist of locally weighted l^p norms of wavelet coefficients calculated over a narrow range of times at all scales less than a given one [12]. The difference from wavelet leaders is the use of the l^p norm instead of l^∞ one.

The Hurst coefficient H – the value of the Hölder coefficient, at which the magnitude of the singularity spectrum reaches its maximum, and the width of this spectrum were chosen as the characteristics to be studied. It should be noted that generalized Hölder coefficients (p exponents) [10,11] are used here. The entire time series was divided into partially overlapping sections of 2048 signals, on each of which the calculation was performed. Partial overlap provides smoother time dependences of the studied quantities.

For RQA analysis, the entire financial time series was divided into partially overlapping sequences of 1024 daily time counts. For these sequences, recurrent matrices of the form were constructed:

$$R_{i,j}(\varepsilon_i, m) = \theta(\varepsilon_i - \|\mathbf{X}_i(m) - \mathbf{X}_j(m)\|), \tag{3}$$

where $\mathbf{X}_i(m)$ is the phase trajectory of a dynamical system in m-dimensional phase space, θ is the Heaviside function, and $\|\ \|$ is the vector norm. The recurrent matrix constructed in the form of a plot is a two-dimensional set of points corresponding to relation (3) and consists of black dots for the close (recurrent) points with the values of ones and white dots for the distinct points with the value of zero. The proximity of point i neighbors is determined by the parameter ε_i. The choice of large values of m

– the dimension of the embedding space is necessary in the case of restoring the attractor based on the Takens theorem for scalar series. Since this procedure is not used in this paper, the value was chosen to be $m = 1$. The choice of the threshold value ε_i significantly affects the results; therefore, it is discussed many times, see the review [13]. If one chooses ε_i too large, then the points will mostly close to each other (recurrent), and the graph will be a black field; while if ε_i is too small, then there will be almost no recurrent points. In this work, the value was empirically chosen to be 0.1 for all points. The patterns obtained were observed in a wide range of threshold values. The free CRP Toolbox software package [13] has been used.

The aim of RQA analysis was to calculate the parameter called determinism and to find its time dependence. In order to define this parameter one has first to introduce the recurrence rate parameter RR:

$$RR(\varepsilon_i) = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{ij}(\varepsilon_i, m),$$ (4)

representing the recurrent points density on the plot. The determinism value is introduced as the ratio of the number of recurrent points forming diagonal lines parallel to the main diagonal to the total number of recurrent points:

$$DET = \frac{\sum_{i,j=1}^{N} D_{ij}}{\sum_{i,j=1}^{N} R_{ij}}$$ (5)

Chaotic, weakly correlated processes either do not have diagonal lines at all, or these lines are very short, while deterministic processes have longer diagonals and fewer isolated recurrent points [13].

4. Results

Results on the Hurst coefficient calculation are shown in figure 1a. All dependencies show a clear minimum that accompanies the financial crisis of 2008. Figure 1b shows the calculated time dependences of the singularity spectrum widths w. This data was obtained for the case of *p=1*. It can be seen that the financial crisis is preceded by a decrease in w thus demonstrating the tendency to monofractalization. For the Nasdaq-100 index this trend is less pronounced. Perhaps this is because the Nasdaq-100 index does not include the financial companies most exposed to stock changes. The close prices, shown in figure 1 were obtained from [14].

Note that a decrease in the spectrum width and Hurst coefficient is also observed for the latest data processed. This suggests that they reflect the beginning for the new ongoing financial crisis. Multifractal methods require time series of considerable length so that they can be analyzed over a wide range of scale levels. Significantly shorter series lengths are required for Recurrent Plots (RP) and Quantitative Reccurent Analysis (RQA).
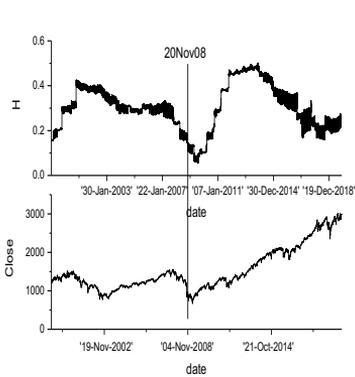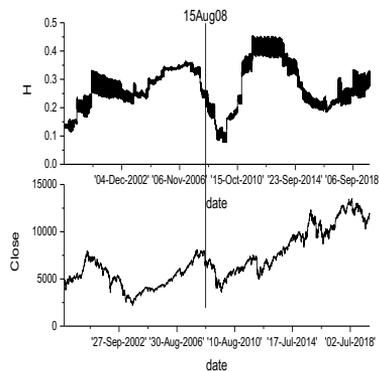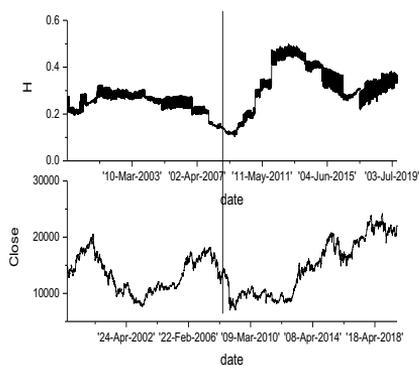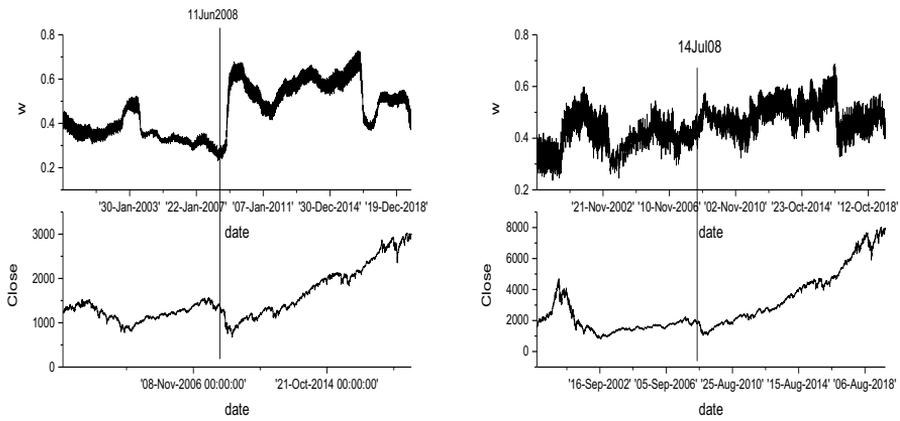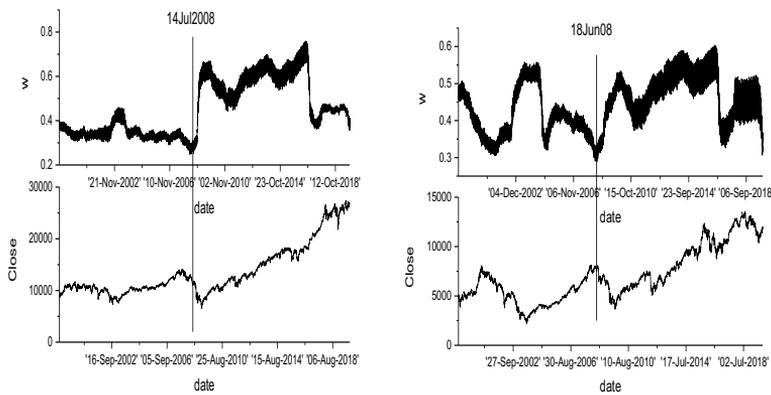
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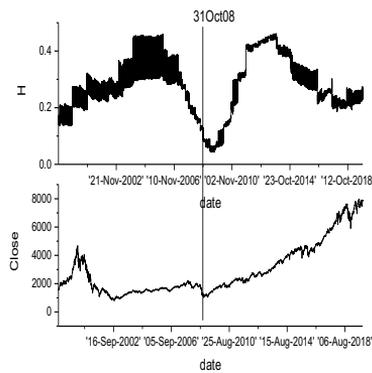
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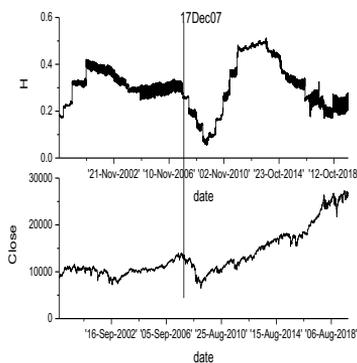
DJI

DAX

Nikkei225

Fig. 1a. Calculated time dependencies of the Hurst coefficients of stock indices and close prices of these indices obtained from [14].
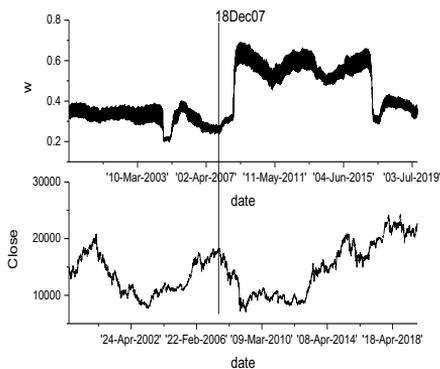
Fig. 1b. Calculated time dependencies of the SS widths of stock indices and close prices of these indices obtained from [14].

Fig. 2 shows the time dependences of the determinism value (DET) for the considered exchange indices calculated using equation (5). It can be noted that the financial crisis of 2008 is accompanied by an increase in determinism. There is also an increase in determinism, starting from 2017 until the end of the studied time range, which, along with the results of multifractal analysis, indicates the possible preparation of a new financial crisis.
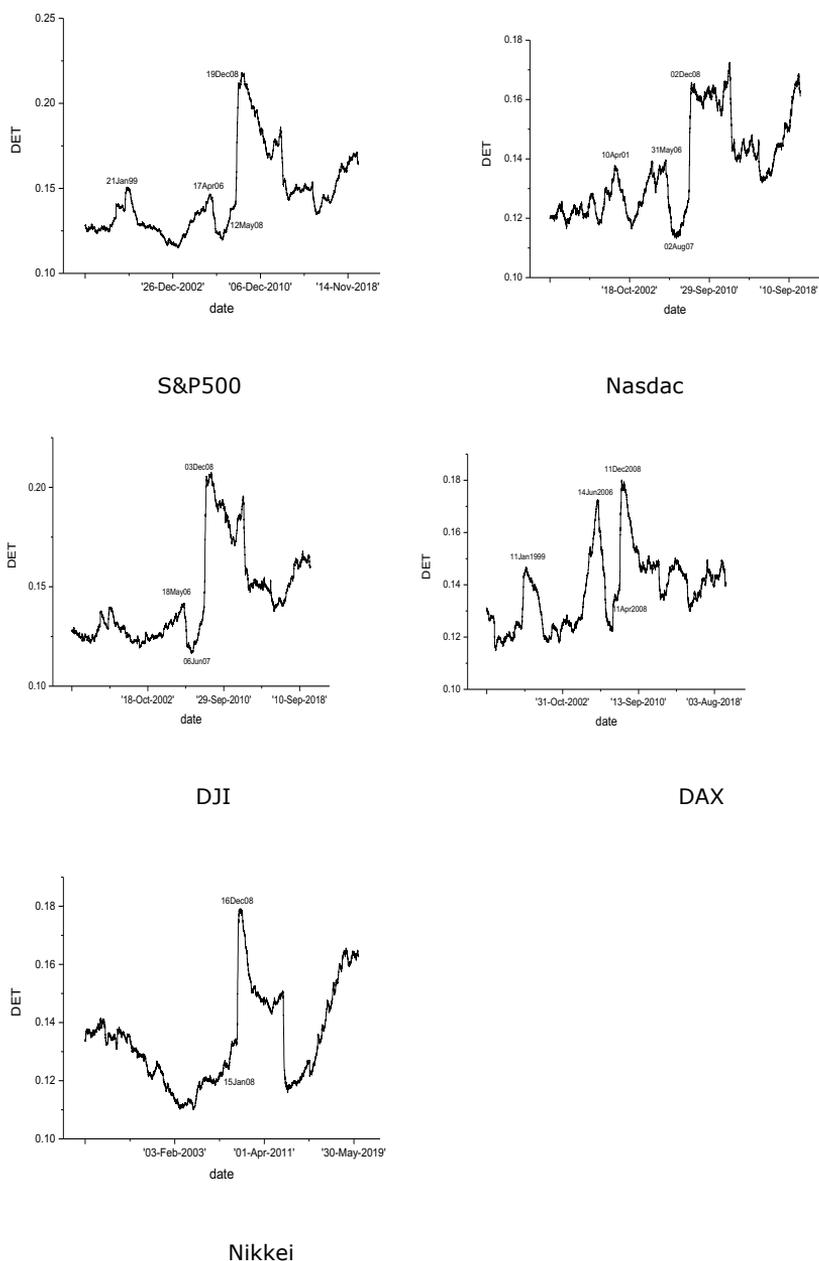

S&P500

Nasdac

DJI

DAX

Nikkei

Fig. 2. Calculated time dependencies of the DET parameter for stock indices.

5. Conclusion

We performed multifractal and recurrence quantification analysis of financial time series in order to investigate fractal self-organization. We can conclude that financial catastrophes, based on the changes in the parameters of daily returns over time, are similar in some respects to natural ones: they are accompanied by a decrease in the width of the singularity spectra and an increase in determinism, demonstrating the tendency to monofractalization. At the same time, in contrast to natural disasters, the Hurst coefficient in financial catastrophes has an antipersistent minimum and turns out to be significantly less than its characteristic value for an efficient market *(0.5).* This is a further confirmation of the fact that the observed fractal self-organization in the systems of quite different nature is not determined by the details of the system elements interaction but rather with the character of these elements collectivization. Besides, the obtained results can be used for the purpose of predicting financial crises

The presence of other extremes on the charts requires further economic justification.

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